

Exam II, MTH 221, Spring 2015

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QUESTION 1. (10 points) Let $A = \begin{bmatrix} 1 & b & 4 \\ a & 3 & 1 \\ 4 & c & 0 \end{bmatrix}$. Given A is row-equivalent to $\begin{bmatrix} 0 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(i) Find the values of b, a, c. **Trivial/ Ideas discussed in class**

(ii) Find a basis for the column space of A. **Trivial**

QUESTION 2. (10 points) Let $S = \{(a + b + 2c, 3a + 6c, 2a - b + 4c) \mid b, c \in R\}$. Is S a subspace of R^3? explain. If yes, find dim(A), find basis for A, and write A as a span of a basis. **Trivial/ basic question**

QUESTION 3. (10 points)

(i) Find a basis for P_4 such that each element in the basis is of degree 3. Show the work. **some thinking is involved here, so we need 4 INDEPENDENT polynomials each is of degree 3. As you translate to points in R^4 (assume polynomials are written in descending order according to their degree), so we need to form a matrix 4×4 such that all entries in the first column are 1 (to ensure getting polynomials each of degree 3). Now you stare at the matrix and choose the other entries so that when you change it to semi-echelon all rows survive. For example Take the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. Now by staring at A, $det(A) \neq 0$. So all rows are independent (or change it to semi-echelon/ all rows will survive). Now translate back to polynomials: so $x^3, x^3 + x^2, x^3 + x, x^3 + 1$ is the desired basis**.

(ii) Let f_1, f_2, f_3, f_4 be polynomial in P_4 such that each is of degree 2. Show that f_1, f_2, f_3, f_4 are dependent.

This is supposed to be a trivial one, note that $f_1, ..., f_4$ are elements of P_3 as well. Since $dim(P_3) = 3$, every 4 elements in P_3 are dependent

QUESTION 4. (12 points) TYPICAL BASIC QUESTION/ SEE CLASS NOTES

a) Let $A = \{F \in R^{3\times 3} \mid Rank(F) \leq 2\}$. Then A is not a subspace of $R^{3\times 3}$. Why?

b) Let $A = \{(a, b, c) \mid a + 2b + c = 4\}$. Then A is not a subspace of R^3. Why?

c) Let $A = \{f(x) \in P_4 \mid f(2) = 0 \text{ or } f(3) = 0\}$. Then A is not a subspace of P_4. Why?

d) Let $A = \{F \in R^{3\times 3} \mid det(A) = 0\}$. Then A is not a subspace of $R^{3\times 3}$. Why?

QUESTION 5. a) **(10 points)** Let $F = \{A \in R^{2\times 2}$ such that $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$. Show that F is a subspace of $R^{2\times 2}$. Then find a basis for F. **Done in class/ see your notes**

b) **(6 points)** Given L = $\{A, B, C, D\}$ is a basis for $R^{2\times 2}$, where $A = \begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -4 \\ 2 & -5 \end{bmatrix}$. Find C and D. Note that C and D are not unique. Show the work

Here is the idea: We need to form a matrix F, 4×4, with 4 independent rows. Of course the first two rows are A and B. Now stare at F add two more rows so that when you change F to the semi-echelon all rows survive. After you do that, translate each row of the two rows you added to an 2×2 matrix.

QUESTION 6. (12 points) Given A is a 2×2 matrix such that $3, -3$ are eigenvalues of A, $E_3 = span\{(4,2)\}$, and $E_{-3} = span\{(-2,0)\}$.

(i) Find the trace of A^{-1}. **basic/ see class notes**

(ii) Show that A^2 is a diagonalizable, i.e., find an invertible matrix W and a diagonal matrix D such that $W^{-1}A^2W = D$. **basic**

(iii) Show that A^T is diagnolizable, i.e., find an invertible matrix W and a diagonal matrix D such that $W^{-1}A^TW = D$. **Since** $F^{-1}AF = D$**,** $F^TA^T(F^{-1})^T = D^T$**. Now here** $W = (F^{-1})^T$**.**

(iv) Find a nonzero 2×4 matrix D such that $AD = 3D$. **by matrix multiplication, let** d_1, d_2, d_3, d_4 **be the columns of** D**. Then** $Ad_1 = 3d_1$**,** $Ad_2 = 3d_2$**, ...,** $Ad_4 = 3d_4$**. By staring at the question and knowing 3 is an eigenvalue of** A**/ all these columns must come from** E_3**. Choose all column of** D **from** E_3 **and we are done.**

QUESTION 7. (18 points) Let A be a 3×3 matrix such that $C_A(x) = x(x-5)^2$. Given $Nul(A) = \{(x_1, 2x_1, 0) | x_1 \in R\}$ and $Nul(5I_3 - A) = \{(3x_3, 0, x_3) | x_3 \in R\}$,

(i) find $det(A)$.

(ii) Is A diagnolizable? if yes, find a diagonal matrix D and an invertible matrix W such that $W^{-1}AW = D$. If no, explain. **No. Since** $dim(E_5) = 1$ **but** 5 **has multiplicaty 2. Note** $Nul(A) = Nul(-A) = E_0$ **and** $Nul(5I_3 - A) = E_5$

(iii) Find Trace of A. **Adding eigenvalues with multiplicaty. So 5 + 5 + 0 = 10**

(iv) Find $det(A - 2I_3)$. **We know eigenvalues of** $A - 2I_3$ **are 0 - 2, 5 - 2, 5 - 2. Multiply them we get** -18

(v) Let $D = A + 11I_3$. Find a nonzero point in R^3, say $v = (a, b, c)$, such that $D \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 11a \\ 11b \\ 11c \end{bmatrix}$ **Since** 0 **is an egivalue of** A **choose a nonzero point** v **in Nul(A) =** E_0**. Since** $Av = 0v = 0$**, we have** $Dv = (A + 11I_3)v = Av + 11v = 0 + 11v = 11v$**.**

(vi) Given $F = \begin{bmatrix} a & b & \pi \\ c & d & e \\ \sqrt{3} & 8 & f \end{bmatrix}$ such that $AF = 5F$. Find all entries of F. What is the rank of F. **The same idea as in V. So the column of** F **must come from** E_5**. So set the first column of** F**,** $(a, c, \sqrt{3}) = (3x_3, 0, x_3)$**. We get** $x_3 = \sqrt{3}$**, c = 0 and** $a = 3x_3 = 3\sqrt{3}$**, now do similar for column II and column III. Since all columns of** F **are coming from** E_5 **and** $dim(E_5) = 1$**, Rank(F) = 1**

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